Cheat Sheet of Machine Learning and Python (and Math) Cheat Sheets

M medium.com/machine-learning-in-practice/cheat-sheet-of-machine-learning-and-python-and-math-cheat-sheets-a4afe4e791b6

There are many facets to Machine Learning. As I started brushing up on the subject, I came across various "cheat sheets" that compactly listed all the key points I needed to know for a given topic. Eventually, I compiled over 20 Machine Learning-related cheat sheets. Some I reference frequently and thought others may benefit from them too. This post contains 27 of the better cheat sheets I've found on the web. Let me know if I'm missing any you like.

Given how rapidly the Machine Learning space is evolving, I imagine these will go out of date quickly, but at least as of June 1, 2017, they are pretty current.

If you want all of the cheat sheets without having to download them individually like I did, <u>I created a zip file containing all 27</u>. Enjoy!

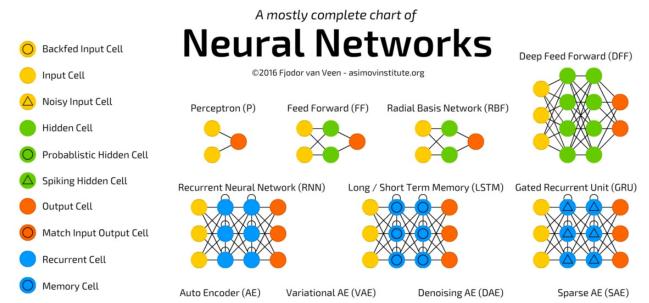
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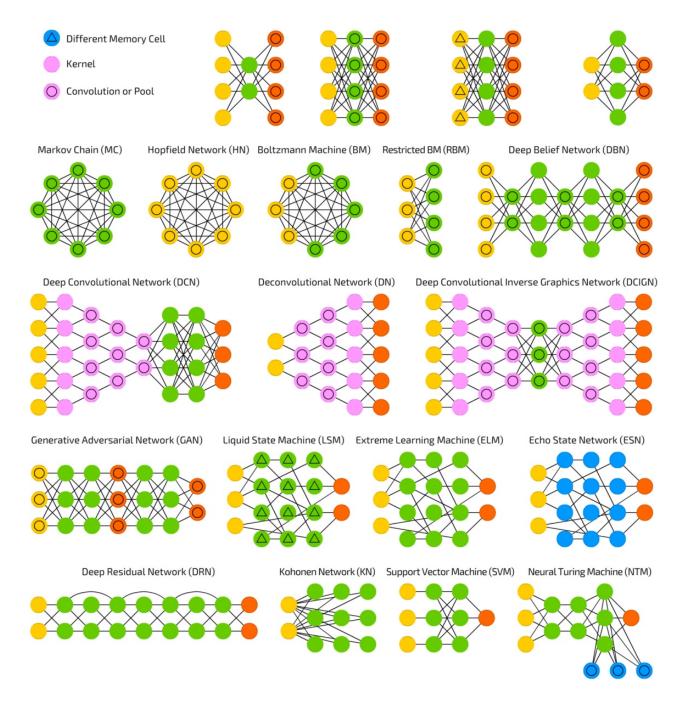
Machine Learning

There are a handful of helpful flowcharts and tables of Machine Learning algorithms. I've included only the most comprehensive ones I've found.

Neural Network Architectures

Source: http://www.asimovinstitute.org/neural-network-zoo/

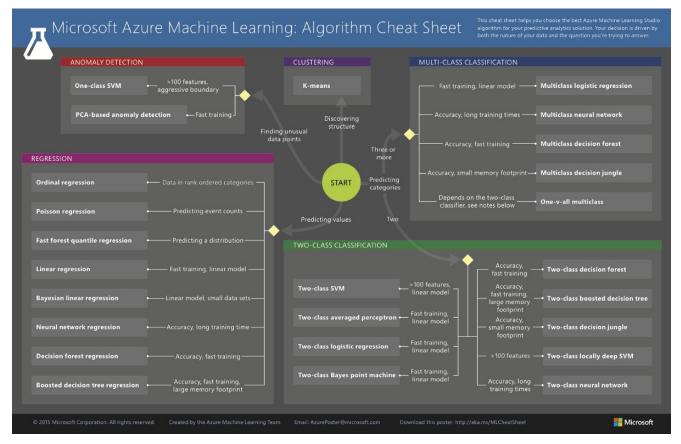




The Neural Network Zoo

Microsoft Azure Algorithm Flowchart

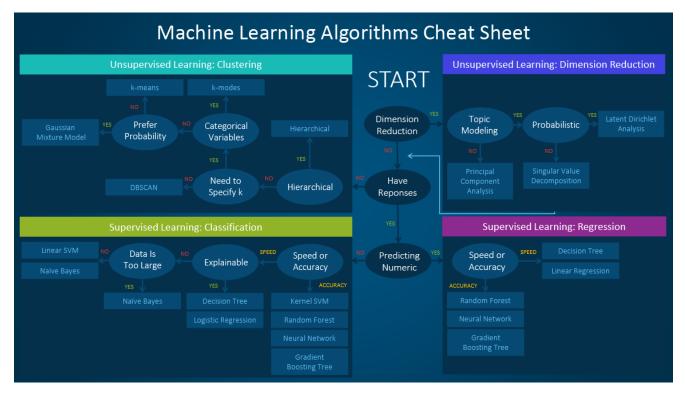
Source: https://docs.microsoft.com/en-us/azure/machine-learning/machine-learning-algorithm-cheat-sheet



Machine learning algorithm cheat sheet for Microsoft Azure Machine Learning Studio

SAS Algorithm Flowchart

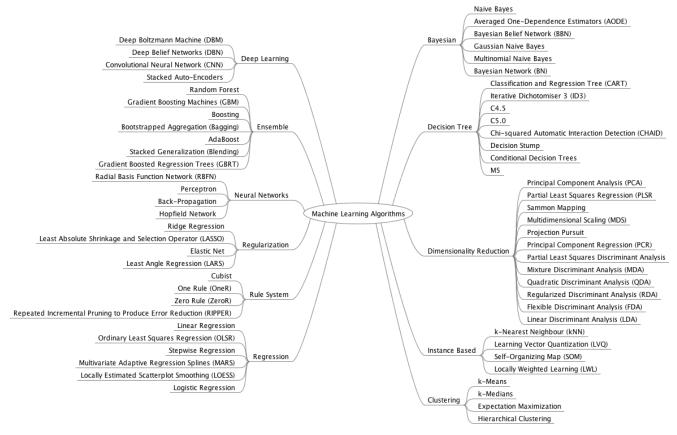
Source: http://blogs.sas.com/content/subconsciousmusings/2017/04/12/machine-learning-algorithm-use/



SAS: Which machine learning algorithm should I use?

Algorithm Summary

Source: http://machinelearningmastery.com/a-tour-of-machine-learning-algorithms/

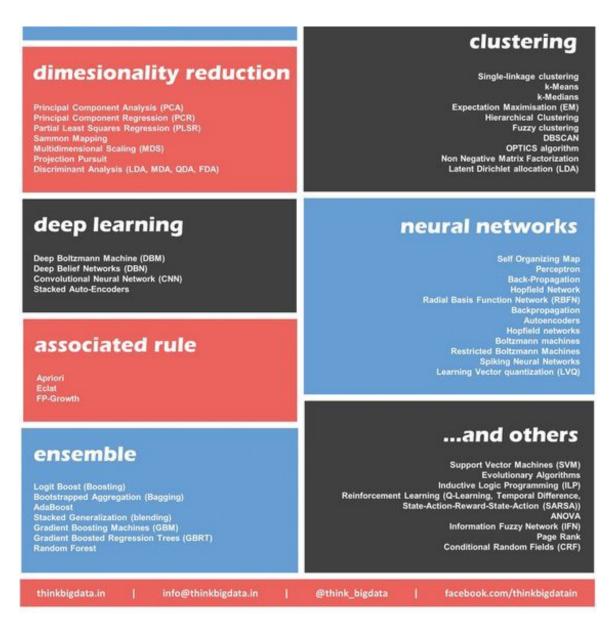


A Tour of Machine Learning Algorithms

Source: http://thinkbigdata.in/best-known-machine-learning-algorithms-infographic/



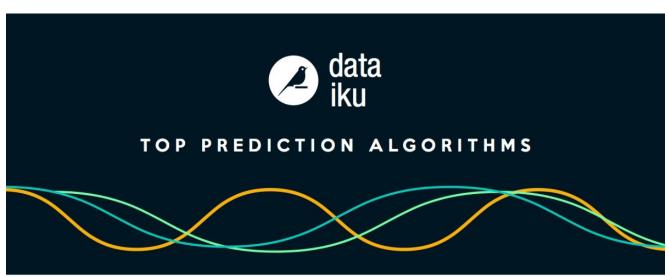




Which are the best known machine learning algorithms?

Algorithm Pro/Con

Source: https://blog.dataiku.com/machine-learning-explained-algorithms-are-your-friend



	ТҮРЕ	NAME	DESCRIPTION	ADVANTAGES	DISADVANTAGES
Linear		Linear regression	The "best fit" line through all data points. Predictions are numerical.	Easy to understand — you clearly see what the biggest drivers of the model are.	Sometimes too simple to capture complex relationships between variables. Tendency for the model to "overfit".
	1	Logistic regression	The adaptation of linear regression to problems of classification (e.g., yes/no questions, groups, etc.)	Also easy to understand.	X Sometimes too simple to capture complex relationships between variables. X Tendency for the model to "overfit".
	\ \.	Decision tree	A graph that uses a branching method to match all possible outcomes of a decision.	Easy to understand and implement.	X Not often used on its own for prediction because it's also often too simple and not powerful enough for complex data.
Tree-based		Random Forest	Takes the average of many decision trees, each of which is made with a sample of the data. Each tree is weaker than a full decision tree, but by combining them we get better overall performance.	A sort of "wisdom of the crowd". Tends to result in very high quality models. Fast to train.	X Can be slow to output predictions relative to other algorithms. X Not easy to understand predictions.
	Y	Gradient Boosting	Uses even weaker decision trees, that are increasingly focused on "hard" examples.	High-performing.	A small change in the feature set or training set can create radical changes in the model. Not easy to understand predictions.
Neural networks	**	Neural networks	Mimics the behavior of the brain. Neural networks are interconnected neurons that pass messages to each other. Deep learning uses several layers of neural networks put one after the other.	Can handle extremely complex tasks - no other algorithm comes close in image recognition.	 Very, very slow to train, because they have so many layers. Require a lot of power. Almost impossible to understand predictions.



Python

Unsurprisingly, there are a lot of online resources available for Python. For this section, I've only included the best cheat sheets I've come across.

Algorithms

Source: https://www.analyticsvidhya.com/blog/2015/09/full-cheatsheet-machine-learning-algorithms/



Machine Learning Algorithms





Supervised Learning

- Decision Tree Random Forest
- ·kNN · Logistic Regression

#Import Library

Unsupervised Learning

- ${}^{\textstyle \cdot} \text{Apriori algorithm} {}^{\textstyle \cdot} \text{k-means}$
- · Hierarchical Clustering

Reinforcement Learning

- · Markov Decision Process
- · Q Learning

Python Code

R Code

gression

#Import other necessary libraries like pandas,
#numpy...
from sklearn import linear_model
#Load Train and Test datasets
#Identify feature and response variable(s) and
#values must be numeric and numpy arrays
x_train=input_variables_values_training_datasets
y_train=target_variables_values_training_datasets
x_test=input_variables_values_test_datasets
#Create linear regression object
linear = linear_model.LinearRegression()
#Train the model using the training sets and
#check score
linear.fit(x_train, y_train)
linear.score(x_train, y_train)

#Load Train and Test datasets
#Identify feature and response variable(s) and
#values must be numeric and numpy arrays
x_train <- input_variables_values_training_datasets
y_train <- target_variables_values_training_datasets
x_test <- input_variables_values_test_datasets
x <- cbind(x_train,y_train)
#Train the model using the training sets and
#check score
linear <- lm(y_train ~ ., data = x)
summary(linear)
#Predict Output
predicted= predict(linear,x_test)</pre>

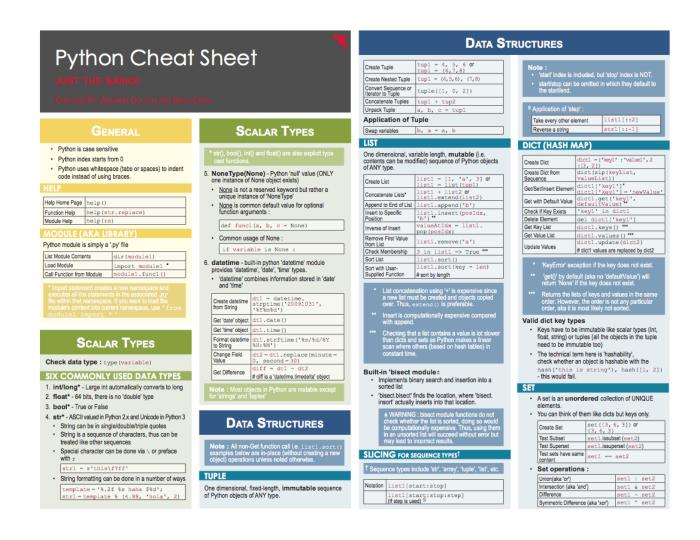
Python Basics

#Predict Output

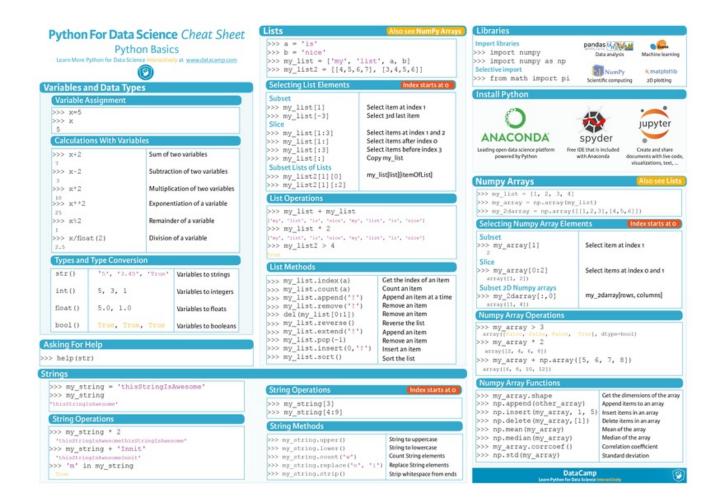
Source: http://datasciencefree.com/python.pdf

#Equation coefficient and Intercept
print('Coefficient: \n', linear.coef_)
print('Intercept: \n', linear.intercept_)

predicted= linear.predict(x_test)



Source: https://www.datacamp.com/community/tutorials/python-data-science-cheat-sheet-basics#gs.0x1rxEA



Numpy

Source: https://www.dataquest.io/blog/numpy-cheat-sheet/

Data Science Cheat Sheet

KEY

We'll use shorthand in this cheat sheet arr - A numpy Array object

IMPORTS

Import these to start import numpy as np

IMPORTING/EXPORTING

np.loadtxt('file.txt') - From a text file

np.genfromtxt('file.csv',delimiter=',')
 -From a CSV file

CREATING ARRAYS

 $\begin{array}{l} & \text{np.array}([1,2,3]) \cdot \text{One dimensional array} \\ & \text{np.array}([(1,2,3),(4,5,6)]) \cdot \text{Two dimensional} \end{array}$

np.zeros(3) - 1D array of length 3 all values 0

np.ones((3,4)) - 3x4 array with all values 1

np.eye(5) - 5x5 array of 0 with 1 on diagonal (Identity matrix)

np.linspace(0,100,6) - Array of 6 evenly divided values from 0 to 100

np.arange(0,10,3) - Array of values from 0 to less than 10 with step 3 (eg [0,3,6,9])

np.full((2,3),8) - 2x3 array with all values 8 np.random.rand(4,5) - 4x5 array of random floats between 0.1

np.random.rand(6,7)*100 - 6x7 array of random floats between 0-100

np.random.randint(5,size=(2,3)) - 2x3 array with random ints between 0-4

INSPECTING PROPERTIES

arr.size - Returns number of elements in arr arr.shape - Returns dimensions of arr (rows,

arr.dtype - Returns type of elements in arr arr.astype(dtype) - Convert arr elements to type dtype

arr.tolist() - Convert arr to a Python list np.info(np.eye) - View documentation for np.eye

COPYING/SORTING/RESHAPING

np.copy(arr) - Copies arr to new memory
arr.view(dtype) - Creates view of arr elements

with type dtype

arr.sort() - Sorts arr

arr.sort(axis=0) - Sorts specific axis of arr two_d_arr.flatten() - Flattens 2D array

two_d_arr to 1D

arr.T - Transposes arr (rows become columns and vice versa)

arr.reshape(3,4) - Reshapes arr to 3 rows, 4 columns without changing data

arr.resize((5,6)) - Changes arr shape to 5x6 and fills new values with 0

ADDING/REMOVING ELEMENTS

np.append(arr,values) - Appends values to end of arr

np.insert(arr,2,values) - Inserts values into arr before index 2

np.delete(arr,3,axis=0) - Deletes row on index 3 of arr

np.delete(arr,4,axis=1) - Deletes column on
index 4 of arr

COMBINING/SPLITTING

np.concatenate((arr1,arr2),axis=0) - Adds
arr2 as rows to the end of arr1

np.concatenate((arr1,arr2),axis=1) - Adds
arr2 as columns to end of arr1

np.split(arr,3) - Splits arr into 3 sub-arrays
np.hsplit(arr,5) - Splits arr horizontally on the
Sth index

INDEXING/SLICING/SUBSETTING

arr[5] - Returns the element at index 5

arr[2,5] - Returns the 2D array element on index
[21[5]

arr[1]=4 - Assigns array element on index 1 the value 4

arr[1,3]=10 - Assigns array element on index
[1][3] the value 10

arr[0:3] - Returns the elements at indices 0,1,2
(On a 2D array: returns rows 0,1,2)

arr[0:3,4] - Returns the elements on rows 0,1,2 at column 4

arr[:2] - Returns the elements at indices θ,1 (On a 2D array: returns rows θ,1)

arr[:,1] - Returns the elements at index 1 on all rows

arr<5 - Returns an array with boolean values (arr1<3) & (arr2>5) - Returns an array with

boolean values
~arr - Inverts a boolean array

arr[arr<5] - Returns array elements smaller than 5

SCALAR MATH

np.add(arr,1) - Add 1 to each array element

np.subtract(arr,2) - Subtract 2 from each array
element

np.multiply(arr,3) - Multiply each array element by 3

np.divide(arr,4) - Divide each array element by 4 (returns np.nan for division by zero)

np.power(arr,5) - Raise each array element to the 5th power

VECTOR MATH

np.add(arr1, arr2) - Elementwise add arr2 to

np.subtract(arr1,arr2) - Elementwise subtract
arr2 from arr1

np.multiply(arr1,arr2) - Elementwise multiply arr1 by arr2

np.divide(arr1, arr2) - Elementwise divide arr1
by arr2

np.power(arr1, arr2) - Elementwise raise arr1 raised to the power of arr2

np.array_equal(arr1,arr2) - Returns True if the arrays have the same elements and shape

np.sqrt(arr) - Square root of each element in the
array

np.sin(arr) - Sine of each element in the array

np.log(arr) - Natural log of each element in the array

np.abs(arr) - Absolute value of each element in the array

np.ceil(arr) - Rounds up to the nearest int np.floor(arr) - Rounds down to the nearest int

np.round(arr) - Rounds to the nearest int

,

STATISTICS

np.mean(arr,axis=0) - Returns mean along
specific axis

arr.sum() - Returns sum of arr

arr.min() - Returns minimum value of arr

arr.max(axis=0) - Returns maximum value of specific axis

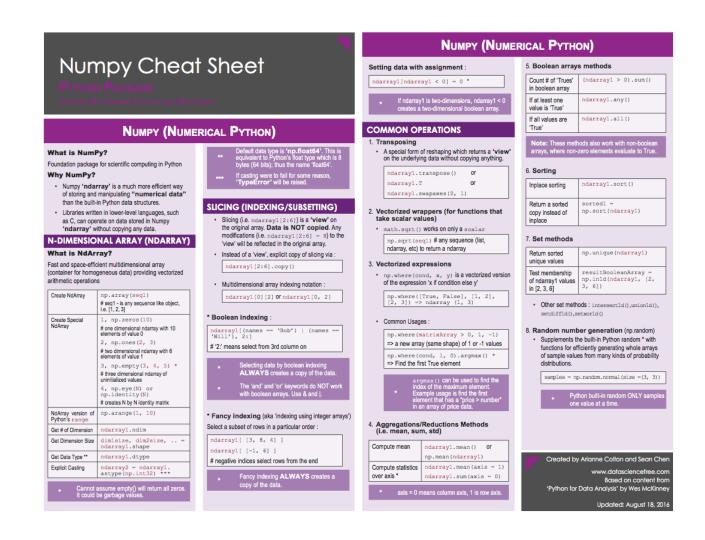
np.var(arr) - Returns the variance of array

np.std(arr,axis=1) - Returns the standard deviation of specific axis

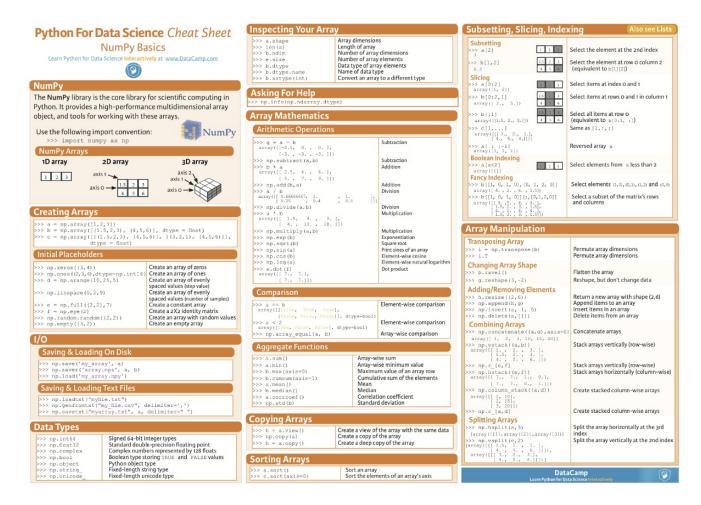
arr.corrcoef() - Returns correlation coefficient of array

LEARN DATA SCIENCE ONLINE Start Learning For Free - www.dataquest.io

Source: http://datasciencefree.com/numpy.pdf



Source: https://www.datacamp.com/community/blog/python-numpy-cheat-sheet#gs.Nw3V6CE



Source: https://github.com/donnemartin/data-science-ipython-notebooks/blob/master/numpy/numpy.ipynb

NumPy

Credits: Forked from Parallel Machine Learning with scikit-learn and IPython by Olivier Grisel

- · NumPy Arrays, dtype, and shape
- · Common Array Operations
- · Reshape and Update In-Place
- · Combine Arrays
- Create Sample Data

```
In [1]: import numpy as np
```

NumPy Arrays, dtypes, and shapes

```
In [2]: a = np.array([1, 2, 3])
    print(a)
    print(a.shape)
    print(a.dtype)

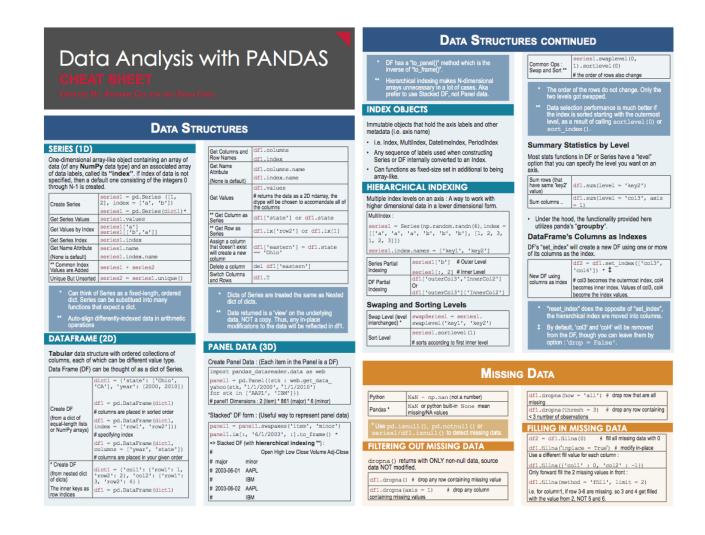
[1 2 3]
    (3,)
    int64

In [3]: b = np.array([[0, 2, 4], [1, 3, 5]])
    print(b)
    print(b.shape)
    print(b.dtype)

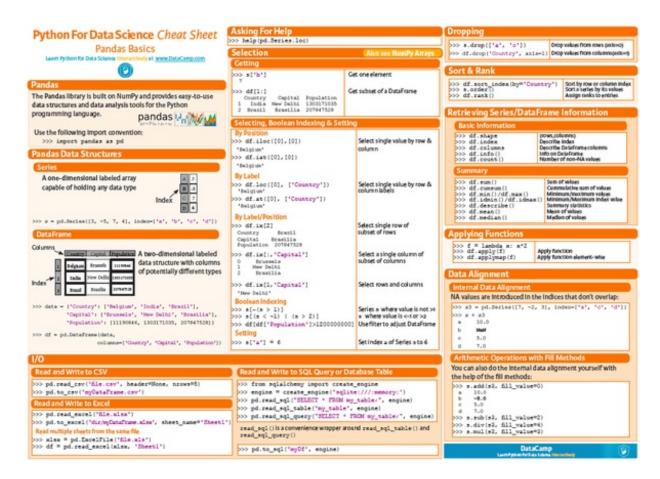
[[0 2 4]
    [1 3 5]]
    (2, 3)
    int64
```

Pandas

Source: http://datasciencefree.com/pandas.pdf



Source: https://www.datacamp.com/community/blog/python-pandas-cheat-sheet#gs.S4P4T=U



Source: https://github.com/donnemartin/data-science-ipython-notebooks/blob/master/pandas/pandas.ipynb

Pandas

Credits: The following are notes taken while working through Python for Data Analysis by Wes McKinney

- Series
- DataFrame
- Reindexing
- · Dropping Entries
- · Indexing, Selecting, Filtering
- · Arithmetic and Data Alignment
- · Function Application and Mapping
- · Sorting and Ranking
- · Axis Indices with Duplicate Values
- · Summarizing and Computing Descriptive Statistics
- · Cleaning Data (Under Construction)
- · Input and Output (Under Construction)

```
In [1]: from pandas import Series, DataFrame import pandas as pd import numpy as np
```

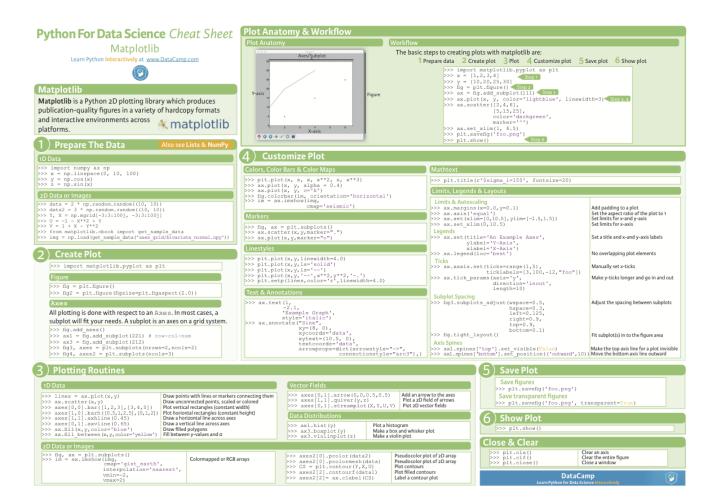
Series

A Series is a one-dimensional array-like object containing an array of data and an associated array of data labels. The data can be any NumPy data type and the labels are the Series' index.

Create a Series:

Matplotlib

Source: https://www.datacamp.com/community/blog/python-matplotlib-cheat-sheet



Source: https://github.com/donnemartin/data-science-ipython-notebooks/blob/master/matplotlib/matplotlib.ipynb

matplotlib

Credits: Content forked from Parallel Machine Learning with scikit-learn and IPython by Olivier Grisel

- · Setting Global Parameters
- · Basic Plots
- Histograms
- · Two Histograms on the Same Plot
- Scatter Plots

```
In [1]: %matplotlib inline
import pandas as pd
import numpy as np
import pylab as plt
import seaborn
```

Setting Global Parameters

```
In [2]: # Set the global default size of matplotlib figures
plt.rc('figure', figsize=(10, 5))

# Set seaborn aesthetic parameters to defaults
seaborn.set()
```

Basic Plots

```
In [3]: x = np.linspace(0, 2, 10)

plt.plot(x, x, 'o-', label='linear')
plt.plot(x, x ** 2, 'x-', label='quadratic')

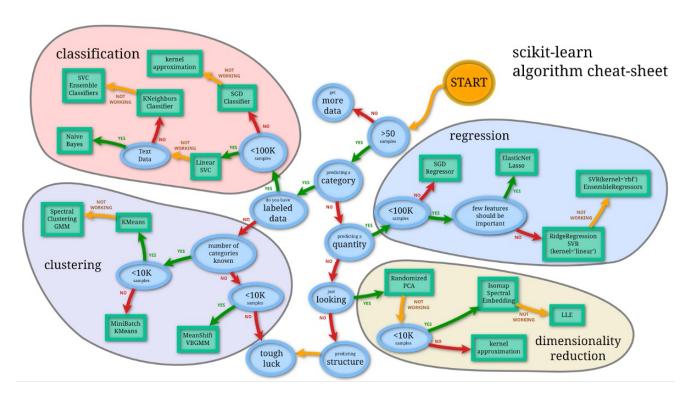
plt.legend(loc='best')
plt.title('Linear vs Quadratic progression')
```

Scikit Learn

Source: https://www.datacamp.com/community/blog/scikit-learn-cheat-sheet#gs.fZ2A1Jk



Source: http://peekaboo-vision.blogspot.de/2013/01/machine-learning-cheat-sheet-for-scikit.html



Source:

https://github.com/rcompton/ml_cheat_sheet/blob/master/supervised_learning.ipynb

```
In [1]: import sklearn
        import numpy as np
        import pandas as pd
        import matplotlib.pyplot as plt
        import seaborn as sns
        import sklearn.datasets
        %pylab inline
        #sklearn two moons generator makes lots of these...
        warnings.filterwarnings("ignore", category=DeprecationWarning)
        Populating the interactive namespace from numpy and matplotlib
In [2]: """
        Build some datasets that I'll demo the models on
        Xs = []
        ys = []
        #low noise, plenty of samples, should be easy
        X0, y0 = sklearn.datasets.make_moons(n_samples=1000, noise=.05)
        Xs.append(X0)
        ys.append(y0)
        #more noise, plenty of samples
        X1, y1 = sklearn.datasets.make moons(n samples=1000, noise=.3)
        Xs.append(X1)
        ys.append(y1)
        #less noise, few samples
        X2, y2 = sklearn.datasets.make moons(n samples=200, noise=.05)
        Xs.append(X2)
        ys.append(y2)
        #more noise, less samples, should be hard
        X3, y3 = sklearn.datasets.make_moons(n_samples=200, noise=.3)
        Xs.append(X3)
```

Tensorflow

Source: https://github.com/aymericdamien/TensorFlow- Examples/blob/master/notebooks/1 Introduction/basic operations.ipynb

```
In [1]: # Basic Operations example using TensorFlow library.
        # Author: Aymeric Damien
        # Project: https://github.com/aymericdamien/TensorFlow-Examples/
In [2]: import tensorflow as tf
In [3]: # Basic constant operations
        # The value returned by the constructor represents the output
        # of the Constant op.
        a = tf.constant(2)
        b = tf.constant(3)
In [4]: # Launch the default graph.
        with tf.Session() as sess:
           print "a: %i" % sess.run(a), "b: %i" % sess.run(b)
            print "Addition with constants: %i" % sess.run(a+b)
            print "Multiplication with constants: %i" % sess.run(a*b)
        a=2, b=3
        Addition with constants: 5
        Multiplication with constants: 6
In [5]: # Basic Operations with variable as graph input
        # The value returned by the constructor represents the output
        # of the Variable op. (define as input when running session)
        # tf Graph input
        a = tf.placeholder(tf.int16)
        b = tf.placeholder(tf.int16)
In [6]: # Define some operations
        add = tf.add(a, b)
        mul = tf.multiply(a, b)
```

Pytorch

Source: https://github.com/bfortuner/pytorch-cheatsheet

Pytorch Cheatsheet

Imports

```
In [1]: import torch
        import torch.nn as nn
        import torch.nn.init as init
        import torch.optim as optim
        import torch.nn.functional as F
        from torch.autograd import Variable
        import torchvision.datasets as datasets
        import torchvision.transforms as transforms
        import torchvision.utils as tv_utils
        from torch.utils.data import DataLoader
        import torchvision.models as models
        import torch.backends.cudnn as cudnn
        import torchvision
        import torch.autograd as autograd
        from PIL import Image
        import imp
        import os
        import sys
        import math
        import time
        import random
        import shutil
        import cv2
        import scipy.misc
        from glob import glob
        import sklearn
        import logging
        from tqdm import tqdm
        import numpy as np
        import matplotlib as mpl
        mpl.use('Agg')
        import matplotlib.pyplot as plt
        plt.style.use('bmh')
        %matplotlib inline
```

Basics

- http://pytorch.org/tutorials/beginner/pytorch_with_examples.html
- http://pytorch.org/tutorials/beginner/blitz/cifar10_tutorial.html

Datasets

File Management

```
In [ ]: random.seed(1)
    torch.manual_seed(1)
    DATA_PATH='/media/bfortuner/bigguy/data/'
```

Math

If you really want to understand Machine Learning, you need a solid understanding of Statistics (especially Probability), Linear Algebra, and some Calculus. I minored in Math during undergrad, but I definitely needed a refresher. These cheat sheets provide most of what you need to understand the Math behind the most common Machine Learning algorithms.

Probability

Source: http://www.wzchen.com/s/probability_cheatsheet.pdf

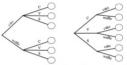
Probability Cheatsheet v2.0

Compiled by William Chen (http://wzchen.com) and Joe Blitzstein, with contributions from Sebastian Chiu, Yuan Jiang, Yuqi Hou, and Jessy Hwang. Material based on Joe Blitzstein's (estati10) lectures (http://stati10.net) and Blitzstein/Hwang's Introduction to Probability textbook (http://bit.ly/introprobability). Licensed under CC BY-NC-SA 4.0. Please share comments, suggestions, and errors at http://github.com/wzchen/probability_cheatsheet.

Last Updated September 4, 2015

Counting

Multiplication Rule



Let's say we have a compound experiment (an experiment with multiple components). If the 1st component has n_1 possible outcomes, the 2nt component has n_2 possible outcomes, ..., and the rth component has n_1 possible outcomes, then overall there are $n_1 n_2 \dots n_t$ possibilities for the whole experiment.

Sampling Table



The sampling table gives the number of possible samples of size k out of a population of size n, under various assumptions about how the sample is collected.

Order Matters	Not Matter
n^k	$\binom{n+k-1}{k}$
n!	$\binom{n}{}$
(n-k)!	(k)
	n^k $n!$

Thinking Conditionally

Independence

Independent Events A and B are independent if knowing whether A occurred gives no information about whether B occurred. More formally, A and B (which have nonzero probability) are independent if and only if one of the following equivalent statements holds:

$$P(A \cap B) = P(A)P(B)$$

$$P(A|B) = P(A)$$

$$P(B|A) = P(B)$$

Conditional Independence A and B are conditionally independent given C if $P(A \cap B|C) = P(A|C)P(B|C)$. Conditional independence does not imply independence, and independence does not imply conditional independence.

Unions, Intersections, and Complements

De Morgan's Laws A useful identity that can make calculating probabilities of unions easier by relating them to intersections, and vice versa. Analogous results hold with more than two sets.

$$(A \cup B)^c = A^c \cap B^c$$
$$(A \cap B)^c = A^c \cup B^c$$

Joint, Marginal, and Conditional

Joint Probability $P(A \cap B)$ or P(A,B) – Probability of A and B. Marginal (Unconditional) Probability P(A) – Probability of A. Conditional Probability P(A|B) = P(A,B)/P(B) – Probability of A, given that B occurred.

Conditional Probability is Probability P(A|B) is a probability function for any fixed B. Any theorem that holds for probability also holds for conditional probability.

Probability of an Intersection or Union

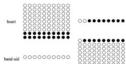
Intersections via Conditioning

$$\begin{split} P(A,B) &= P(A)P(B|A) \\ P(A,B,C) &= P(A)P(B|A)P(C|A,B) \end{split}$$

Unions via Inclusion-Exclusion

$$\begin{split} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ P(A \cup B \cup C) &= P(A) + P(B) + P(C) \\ &\qquad - P(A \cap B) - P(A \cap C) - P(B \cap C) \\ &\qquad + P(A \cap B \cap C). \end{split}$$

Simpson's Paradox



Probability Cheatsheet v2.0

Law of Total Probability (LOTP)

Let $B_1,B_2,B_3,...B_n$ be a partition of the sample space (i.e., they are disjoint and their union is the entire sample space).

$$P(A) = P(A|B_1)P(B_1) + P(A|B_2)P(B_2) + \dots + P(A|B_n)P(B_n)$$

$$P(A) = P(A \cap B_1) + P(A \cap B_2) + \dots + P(A \cap B_n)$$

For LOTP with extra conditioning, just add in another event C!

$$\begin{split} P(A|C) &= P(A|B_1, C)P(B_1|C) + \dots + P(A|B_n, C)P(B_n|C) \\ P(A|C) &= P(A \cap B_1|C) + P(A \cap B_2|C) + \dots + P(A \cap B_n|C) \end{split}$$

Special case of LOTP with B and B^c as partition:

$$P(A) = P(A|B)P(B) + P(A|B^{\circ})P(B^{\circ})$$

$$P(A) = P(A \cap B) + P(A \cap B^{\circ})$$

Bayes' Rule

Bayes' Rule, and with extra conditioning (just add in C!)

$$\begin{split} P(A|B) &= \frac{P(B|A)P(A)}{P(B)} \\ P(A|B,C) &= \frac{P(B|A,C)P(A|C)}{P(B|C)} \end{split}$$

We can also write

$$P(A|B,C) = \frac{P(A,B,C)}{P(B,C)} = \frac{P(B,C|A)P(A)}{P(B,C)}$$

Odds Form of Bayes' Rule

$$\frac{P(A|B)}{P(A^c|B)} = \frac{P(B|A)}{P(B|A^c)} \frac{P(A)}{P(A^c)}$$

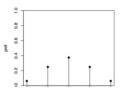
The posterior odds of A are the likelihood ratio times the prior odds.

Random Variables and their Distributions

PMF, CDF, and Independence

Probability Mass Function (PMF) Gives the probability that a discrete random variable takes on the value x.

$$p_X(x) = P(X = x)$$



Linear Algebra

Source: https://minireference.com/static/tutorials/linear_algebra_in_4_pages.pdf

Linear algebra explained in four pages

Excerpt from the NO BULLSHIT GUIDE TO LINEAR ALGEBRA by Ivan Savov

Abstract—This document will review the fundamental ideas of linear algebra. We will learn about matrices, matrix operations, linear transformations and discuss both the theoretical and computational aspects of linear algebra. The tools of linear algebra open the gateway to the study of more advanced mathematics. A lot of knowledge buzz awaits you if you choose to follow the path of understanding, instead of trying to memorize a bunch of formulas.

I. INTRODUCTION

Linear algebra is the math of vectors and matrices. Let n be a positive integer and let \mathbb{R} denote the set of real numbers, then \mathbb{R}^n is the set of all n-tuples of real numbers. A vector $\vec{v} \in \mathbb{R}^n$ is an n-tuple of real numbers. The notation " \in S" is read "element of S." For example, consider a vector that has three components:

$$\vec{v} = (v_1, v_2, v_3) \in (\mathbb{R}, \mathbb{R}, \mathbb{R}) \equiv \mathbb{R}^3.$$

A matrix $A \in \mathbb{R}^{m \times n}$ is a rectangular array of real numbers with m rows and n columns. For example, a 3×2 matrix looks like this:

$$A = \left[\begin{array}{cc} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{array} \right] \; \in \; \left[\begin{array}{cc} \mathbb{R} & \mathbb{R} \\ \mathbb{R} & \mathbb{R} \\ \mathbb{R} & \mathbb{R} \end{array} \right] \equiv \mathbb{R}^{3 \times 2}.$$

The purpose of this document is to introduce you to the mathematical operations that we can perform on vectors and matrices and to give you a feel of the power of linear algebra. Many problems in science, business, and technology can be described in terms of vectors and matrices so it is important that you understand how to work with these.

The only prerequisite for this tutorial is a basic understanding of high school math concepts1 like numbers, variables, equations, and the fundamental arithmetic operations on real numbers: addition (denoted +), subtraction (denoted -), multiplication (denoted implicitly), and division (fractions).

B. Matrix operations

We denote by A the matrix as a whole and refer to its entries as a_{ij} . The mathematical operations defined for matrices are the following:

addition (denoted +)

$$C = A + B \Leftrightarrow c_{ij} = a_{ij} + b_{ij}.$$

- subtraction (the inverse of addition)
- matrix product. The product of matrices $A \in \mathbb{R}^{m \times n}$ and $B \in \mathbb{R}^{n \times \ell}$ is another matrix $C \in \mathbb{R}^{m \times \ell}$ given by the formula

$$C=AB$$
 \Leftrightarrow $c_{ij}=\sum_{k=1}^n a_{ik}b_{kj},$

$$C = AB \qquad \Leftrightarrow \qquad c_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj},$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \\ a_{31}b_{11} + a_{32}b_{21} & a_{31}b_{12} + a_{32}b_{22} \end{bmatrix}$$

- matrix inverse (denoted A^{-1})
- matrix transpose (denoted ^T):

$$\begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \end{bmatrix}^\mathsf{T} = \begin{bmatrix} \alpha_1 & \beta_1 \\ \alpha_2 & \beta_2 \\ \alpha_3 & \beta_3 \end{bmatrix}.$$

- matrix trace: $\text{Tr}[A] \equiv \sum_{i=1}^{n} a_{ii}$
- determinant (denoted det(A) or |A|)

Note that the matrix product is not a commutative operation: $AB \neq BA$.

C. Matrix-vector product

The matrix-vector product is an important special case of the matrixmatrix product. The product of a 3×2 matrix A and the 2×1 column vector \vec{x} results in a 3×1 vector \vec{y} given by:

Linear algebra explained in four pages

Statistics

Source: http://web.mit.edu/~csvoss/Public/usabo/stats handout.pdf

Statistics Cheat Sheet

Population

The entire group one desires information about

Sample

A subset of the population taken because the entire population is usually too large to analyze Its characteristics are taken to be representative of the population

Mean

Also called the arithmetic mean or average

The sum of all the values in the sample divided by the number of values in the sample/population μ is the mean of the population; \overline{x} is the mean of the sample

Median

The value separating the higher half of a sample/population from the lower half

Found by arranging all the values from lowest to highest and taking the middle one (or the mean of the middle two if there are an even number of values)

Variance

Measures dispersion around the mean

Determined by averaging the squared differences of all the values from the mean

Variance of a population is σ^2 Can be calculated by subtracting the square of the mean from the

average of the squared scores:

$$\sigma^2 = \frac{\sum (x - \mu)^2}{n}$$

$$\sigma^2 = \frac{\sum x^2}{n} - \mu^2$$

Statistics Cheat Sheet

Calculus

Source: http://tutorial.math.lamar.edu/getfile.aspx?file=B,41,N

"Working" Definition: We say $\lim_{x \to a} f(x) = L$ if we can make f(x) as close to L as we want by taking x sufficiently close to a (on either side of a) without letting x = a.

Right hand limit: $\lim_{x\to a^+} f(x) = L$. This has the same definition as the limit except it requires x > a.

Left hand limit: $\lim_{x \to a} f(x) = L$. This has the same definition as the limit except it requires

Limit at Infinity: We say $\lim_{x \to a} f(x) = L$ if we can make f(x) as close to L as we want by taking x large enough and positive.

There is a similar definition for $\lim_{x\to x^{-1}} f(x) = L$ except we require x large and negative.

Infinite Limit: We say $\lim_{x \to a} f(x) = \infty$ if we can make f(x) arbitrarily large (and positive) by taking x sufficiently close to a (on either side of a) without letting x = a.

There is a similar definition for $\lim f(x) = -\infty$ except we make f(x) arbitrarily large and negative.

Relationship between the limit and one-sided limits

$$\lim_{x \to a} f(x) = L \implies \lim_{x \to a^-} f(x) = \lim_{x \to a^-} f(x) = L \qquad \lim_{x \to a^-} f(x) = \lim_{x \to a^-} f(x) = L \implies \lim_{x \to a} f(x) = L$$

$$\lim_{x \to a^+} f(x) \neq \lim_{x \to a^-} f(x) \implies \lim_{x \to a^-} f(x) \text{ Does Not Exist}$$

Properties

Assume $\lim f(x)$ and $\lim g(x)$ both exist and c is any number then,

1.
$$\lim_{x\to a} \left[cf(x) \right] = c \lim_{x\to a} f(x)$$

4.
$$\lim_{x \to a} \left[\frac{f(x)}{g(x)} \right] = \lim_{x \to a} \frac{f(x)}{g(x)} \text{ provided } \lim_{x \to a} g(x) \neq 0$$

2.
$$\lim_{x \to a} \left[f(x) \pm g(x) \right] = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)$$
5.
$$\lim_{x \to a} \left[f(x) \right]^{a} = \left[\lim_{x \to a} f(x) \right]^{a}$$

5.
$$\lim_{x \to a} [f(x)]^n = \left[\lim_{x \to a} f(x)\right]$$

3.
$$\lim_{x \to a} \left[f(x) g(x) \right] = \lim_{x \to a} f(x) \lim_{x \to a} g(x)$$
6.
$$\lim_{x \to a} \left[\sqrt[a]{f(x)} \right] = \sqrt[a]{\lim_{x \to a} f(x)}$$

Basic Limit Evaluations at $\pm \infty$

Note: $\operatorname{sgn}(a) = 1$ if a > 0 and $\operatorname{sgn}(a) = -1$ if a < 0.

1.
$$\lim_{x\to\infty} \mathbf{e}^x = \infty$$
 & $\lim_{x\to-\infty} \mathbf{e}^x = 0$

5.
$$n \text{ even} : \lim_{r \to +\infty} x^n = \infty$$

Calculus Cheat Sheet

Evaluation Techniques L'Hospital's Rule

If
$$\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{1}{0}$$
 or $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\pm \infty}{\pm \infty}$ then,

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)} \text{ a is a number, } \infty \text{ or } -\infty$$

Polynomials at Infinity

p(x) and q(x) are polynomials. To compute $\lim_{x \to \pm x} \frac{p(x)}{q(x)}$ factor largest power of x in q(x) out

of both p(x) and q(x) then compute limit.

$$\lim_{s \to \infty} \frac{3x^2 - 4}{5x - 2x^2} = \lim_{s \to \infty} \frac{x^2 \left(3 - \frac{4}{x^2}\right)}{x^2 \left(\frac{5}{5} - 2\right)} = \lim_{s \to \infty} \frac{3 - \frac{4}{x^2}}{\frac{5}{5} - 2} = -\frac{3}{2}$$

Piecewise Function

$$\lim_{x \to -2} g(x) \text{ where } g(x) = \begin{cases} x^2 + 5 & \text{if } x < -2 \\ 1 - 3x & \text{if } x \ge -2 \end{cases}$$

Compute two one sided limits,

$$\lim_{x \to -2^{-}} g(x) = \lim_{x \to -2^{-}} x^{2} + 5 = 9$$

$$\lim_{x \to -2^+} g(x) = \lim_{x \to -2^+} 1 - 3x = 7$$

One sided limits are different so $\lim g(x)$ doesn't exist. If the two one sided limits had been equal then $\lim_{x\to x^2} g(x)$ would have existed and had the same value.

Continuous Functions

Factor and Cancel

If f(x) is continuous at a then $\lim_{x \to a} f(x) = f(a)$

f(x) is continuous at b and $\lim_{x \to a} g(x) = b$ then

 $= \lim_{x \to 2} \frac{x+6}{x} = \frac{8}{2} = 4$

 $= \lim_{x \to 9} \frac{9 - x}{\left(x^2 - 81\right)\left(3 + \sqrt{x}\right)} = \lim_{x \to 9} \frac{-1}{\left(x + 9\right)\left(3 + \sqrt{x}\right)}$

 $= \lim_{h \to 0} \frac{1}{h} \left(\frac{-h}{x(x+h)} \right) = \lim_{h \to 0} \frac{-1}{x(x+h)} = -\frac{1}{x^2}$

Continuous Functions and Composition

 $\lim_{x \to a} f(g(x)) = f(\lim_{x \to a} g(x)) = f(b)$

 $\lim_{x \to 2} \frac{x^2 + 4x - 12}{x^2 - 2x} = \lim_{x \to 2} \frac{(x - 2)(x + 6)}{x(x - 2)}$

Rationalize Numerator/Denominator

 $\lim_{x \to 9} \frac{3 - \sqrt{x}}{x^2 - 81} = \lim_{x \to 9} \frac{3 - \sqrt{x}}{x^2 - 81} \frac{3 + \sqrt{x}}{3 + \sqrt{x}}$

9-x

Combine Rational Expressions

 $\lim_{h\to 0}\frac{1}{h}\left(\frac{1}{x+h}-\frac{1}{x}\right)=\lim_{h\to 0}\frac{1}{h}\left(\frac{x-(x+h)}{x(x+h)}\right)$

 $=\frac{-1}{(18)(6)}=-\frac{1}{108}$

- Polynomials for all x.
 Rational function, except for x's that give division by zero.
- 3. $\sqrt[n]{x}$ (n odd) for all x.
- 4. $\sqrt[n]{x}$ (n even) for all $x \ge 0$.
- 5. e^x for all x.
- 6. $\ln x$ for x > 0.
- 7. cos(x) and sin(x) for all x.
- 8. tan(x) and sec(x) provided
- $x \neq \dots, -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \dots$
- 9. cot(x) and csc(x) provided $x \neq \cdots, -2\pi, -\pi, 0, \pi, 2\pi, \cdots$

Calculus Cheat Sheet